Analysis of a three-dimensional chaotic system and its FPGA implementation

Hefei Li¹, Xianghui Hu¹

¹ Tianjin University of Science and Technology,
1038 Daguanlu Road, Hexi District, Tianjin, 300222, China
E-mail: 1027755628@qq.com
www.tust.edu.cn

Abstract
In this letter a three-dimensional chaotic system is implemented based on Field Programmable Gate Array (FPGA). The 3-D chaotic system has a very complex chaotic characteristic with its a real four-wing chaotic attractor. By means of numerical simulation, phase orbits, bifurcation diagram and Lyapunov exponents are given and analyzed to observe dynamic characteristics of the three-dimensional chaotic system. Numerical simulation and the results of implementation in FPGA show that this chaotic system really has many obvious characteristics of chaos.

Keywords: three-dimensional chaotic system, bifurcation diagram Lyapunov exponents, FPGA

1. Introduction
Lorenz system¹ was proposed as the first chaotic system in year 1963, which created a precedent for the study of chaotic systems. Later, scholars and researchers began to do a lot of research on the chaotic system and found and put forward many new chaotic systems. Such as Chen system², Lü system³, Chua system⁴ and so on. The research on chaotic characteristics can be used in many application areas, such secure communication⁵, image encryption⁶ and etc.

Lü system was proposed in year 2002 by Lü et.al to be a novel chaotic system with its unique chaotic characteristics different from other chaotic systems. After then, many research began to be done on Lü system. A hybrid TS fuzzy modeling was proposed approach for the newly coined chaotic Lü system in⁷. A new hyper chaotic attractor coined from the chaotic Lü system was reported by using a state feedback controller, and theoretical analyses and simulation experiments are conducted to investigate the dynamical behavior of the proposed hyper chaotic system in⁸. A new three-dimensional chaotic system, named as generalized augmented Lü system, was proposed in⁹ by using the method of anti-control chaos.

In this paper, the chaotic characteristics of the integer order generalized augmented Lü system is analyzed by means of phase orbits, bifurcation diagram and Lyapunov exponents. The uniqueness of this chaotic system is that it implements a real four-wing attractor with compound structure and contains two mirrored symmetrical subsystem. A circuit is designed based on DSP Builder in FPGA to implement and observe the real four-wing chaotic attractor. Numerical simulation and the results of implementation in FPGA show that this chaotic system really has many obvious characteristics of chaos.

2. Analysis of the generalized augmented Lü system
The dynamical system of the generalized augmented Lü system is described as following,

\[
\begin{align*}
\dot{x} &= -\frac{ab}{(a + b)}x - yz \\
\dot{y} &= ay + xz \\
\dot{z} &= bz + xy + cx
\end{align*}
\]

(1)
Where the system parameters $a$, $b$ are negative real constants, and $c$ is a real constant.

2.1 Chaotic attractor of the system

For the system (1), when the parameters $a = -10$, $b = -4$, $c = 1$ and the initial state $[1, 2, 3]^T$, it is chaotic and has a real three dimensional four-wing chaotic attractor as shown in Fig.1.(1)and Fig.1.(2).

![Fig.1. (1) x-y plane](image1)

![Fig.1. (2) x-z plane](image2)

Fig.1 a real four-wing chaotic attractor of system (1): the parameters $a = -10$, $b = -4$, $c = 1$ and the initial state $[1, 2, 3]^T$.

2.2 Dynamics analysis of the system

Bifurcation diagram and Lyapunov exponents can well reflect the dynamic characteristics of the system. So we mainly observe the Bifurcation diagram and Lyapunov exponents of the system to analyze the dynamic behavior of the system. As shown in the following Figs, the Bifurcation diagram and Lyapunov exponents are given varying the parameters $a$ or $b$.

![Fig.2(1) Lyapunov exponents](image3)

![Fig.2(2) Bifurcation diagram](image4)

Fig.2 Lyapunov exponents and Bifurcation diagram of system (1): parameters value $a = -10$, $c = 1$, $b \in [-8, 0]$ and the initial state $[1, 2, 3]^T$.

![Fig.3(1) Lyapunov exponents](image5)

![Fig.3(2) Bifurcation diagram](image6)

Fig.3 Lyapunov exponents and Bifurcation diagram of system (1): parameters $a = -10$, $b = -4$, $c \in [-4, 4]$ and the initial state $[1, 2, 3]^T$.

From the Lyapunov exponents shown in Fig.2(1) and Fig.3(1), there is only a single positive Lyapunov within a certain interval respectively, which shows that the system is chaotic. The corresponding Bifurcation diagram also indicates the system exists chaos. Especially, as shown in Fig.2(1), the Lyapunov exponents is almost symmetrical on the origin. In fact, when we set $a = -10$, $b = -4$, $c = -1$, a four-wing chaotic attractor can also be obtained and it has the same topological structure with the chaotic attractor shown in Fig. 1. But the difference is that it mirrored changes in the direction of the corresponding plane through the chaotic trajectory.

3. FPGA realization of chaotic attractor

In this paper, the method of Field Programmable Gate Array is applied to implement the chaotic attractor of system (1). This is a method similar to analog circuits that the chaotic system can be shown in oscilloscope. A chaotic circuit is designed by applying the DSP Builder module based on the system (1) as shown in Fig. 4. And then, the chaotic circuit is derived as V-HDL language that will be download to FPGA development board. Here, the main chip of the FPGA development board is EP3C25Q240C8N. The adders, delays, multipliers, amplifiers and data selectors in circuit come from the DSP Builder module and the digital integrator is designed in Fig.4(1). As shown in Fig.4(2), the obtained discrete digital signal is converted to analog signal by high speed D/A, and two of the three singles are inputted in the oscilloscope. By this method, we can obtain the chaotic attractor shown in Fig.5(1) and Fig.5(2) which are the same as the chaotic attractor shown in Fig.1(1) and Fig.1(2).
4. Conclusion

In this paper, the chaotic characteristics and dynamic behaviors of a three-dimensional chaotic system is analyzed by the method of bifurcation diagram and Lyapunov exponents. Theoretical analysis and numerical simulation are applied to analysis the characteristics of the generalized augmented Lü system. In the last section, a method is applied based on FPGA and EDA tool to realize the chaotic attractor of the generalized augmented Lü system. The numerical simulation results and experimental results are completely consistent, showing that this method is practical and can be used in other chaotic system.

References