Three-Dimensional Leader-Follower Formation Flocking of Multi-Agent System

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Abstract
This paper aims to investigate the three-dimensional formation flocking of multi-agent system with leader-follower structure. Based on the nearest neighbor interaction rules, a kind of distributed control algorithm is proposed, which enables all the agents asymptotically converge to fly with the same velocity and approach the expected formation with their neighbors, provided that the initial interaction network of the system is leader-follower connected. Numerical simulations are carried on the multi-agent system to validate the functionality of the proposed algorithm.

Keywords: Formation control, flocking cooperation, leader-follower structure, multi-agent system.

1. Introduction
Formation control of multi-agent system has received more and more attention from the scholars all over the world, due to its widespread applications on military and civilian areas. Except for these traditional methods, such as leader-follower [1]-[2], behavior-based [3]-[4], virtual structure [5]-[7], new algorithms are investigated to give some new results on formation control problem [8]-[9]. In this paper, we try to apply the flocking strategy to solve the formation control problem of multi-agent system in this paper.

Flocking is such a collective phenomenon that can be widely observed from the social animals in nature, and it is also a kind of typical distributed behavior worth to be investigated in further [10]-[13]. For example, the wild geese fly as a herringbone three-dimensional formation when they migrate from one place to another. Therein, the leading wild goose plays an important role. We introduce leader-follower structure to the multi-agent system for the purpose of simulating the formation behavior of the wild geese. Thus, in this paper, we aim to investigate the three-dimensional leader-follower formation flocking problem of multi-agent system.

We consider the multi-agent system consists of one leader and several followers. Each agent is modeled as an extended second-order unicycle with limited interaction capability. Graph theory is used to depict the interaction network of the multi-agent system. A distributed algorithm is proposed with the combination of consensus algorithm and artificial potential field method. The numerical simulation based on MATLAB further validates the functionality of the proposed algorithm for a heterogeneous multi-agent system with one leader and nine followers.

The rest of this paper is organized as follows. The model of each agent is given and the formation flocking problem is formulated in section II. Section III presents a distributed control algorithm to solve the formation flocking problem. In section IV, the simulation results are shown by six agents. Finally, the conclusions are drawn in section V.

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2. Model and Problem Statement

Firstly, we model each agent by the following kinematic equation

\[
\begin{align*}
\dot{x}_i(t) &= v_i(t)\cos\theta_i(t)\cos\phi_i(t) - a_\phi(t)\sin\theta_i(t), \\
\dot{y}_i(t) &= v_i(t)\sin\theta_i(t), \\
\dot{z}_i(t) &= v_i(t)\cos\theta_i(t)\sin\phi_i(t) + a_\phi(t)\cos\theta_i(t), \\
\dot{\theta}_i(t) &= 0, \\
\dot{v}_i(t) &= a_i(t) + \alpha_i(t), \\
\dot{a}_i(t) &= b_i(t)/\ell_i, \\
\dot{\phi}_i(t) &= c_i(t)
\end{align*}
\]

where \( p_i(t) = [x_i(t), y_i(t), z_i(t)]^T \in \mathbb{R}^3 \) is the position vector of the agent \( i \) at time \( t \), \( \theta_i \in [0, 2\pi) \) is the yaw angle of the agent \( i \) at time \( t \), \( \phi_i \in (-\pi/2, \pi/2) \) is the pitch angle of the agent \( i \) at time \( t \), \( a_i(t) = [a_i(t), a_i\phi(t)]^T = [v_i(t), \dot{\theta}_i(t)]^T \) is the velocity vector of the agent \( i \) at time \( t \), \( v_i(t) \in \mathbb{R} \) is the thrusting speed of the agent \( i \) at time \( t \), \( a_i(t) \in \mathbb{R} \) is the rotational speed of the agent \( i \) at time \( t \), \( \ell_i \) is the distance between the geometrical center \( C_i \) and the mass center \( M_i \) of the agent \( i \), \( \dot{\theta}_i(t) = a_i(t)/\ell_i \) is the tangential speed of the agent \( i \) at time \( t \), \( a_i(t) \in \mathbb{R} \) is the thrusting acceleration of the agent \( i \) at time \( t \), \( b_i(t) \in \mathbb{R} \) is the rotational acceleration of the agent \( i \) at time \( t \), and \( c_i(t) \in \mathbb{R} \) is the pitching acceleration of the agent \( i \) at time \( t \). \( \ell_i \) is a positive constant, \( i = 1, \ldots, N \).

With \( p_i(t) = [x_i(t), y_i(t), z_i(t)]^T \) and \( q_i(t) = [v_i(t), a_i(t)]^T \), some equations in (1) can be merged as the following matrix form

\[
\begin{align*}
\dot{p}_i(t) &= H_i(t)q_i(t), \\
\dot{q}_i(t) &= u_i(t),
\end{align*}
\]

where

\[
H_i(t) = [v_i(t), a_i(t)]^T = \begin{bmatrix} \cos\phi_i(t)\cos\theta_i(t) & \cos\phi_i(t)\sin\theta_i(t) \\ -\sin\phi_i(t)\sin\theta_i(t) & \cos\phi_i(t)\cos\theta_i(t) \end{bmatrix}
\]

and \( u_i(t) = [a_i(t), b_i(t)]^T \). Besides, \( u_i(t) = [a_i(t), b_i(t), c_i(t)]^T \) is taken as the control input of the agent \( i \).

The multi-agent system under consideration consists of one leader and \( N-1 \) followers. Let leader set be \( L = \{1\} \) and follower set be \( F = \{2, \ldots, N\} \). If an agent is dominated by external control input, we call it a leader; otherwise, we call it a follower. Let \( N_i(t) \) denote the neighbor set of the follower \( i \in F \) at time \( t \), and the initial neighbor set of the follower \( i \) is defined as

\[
N_i(0) = \{j \mid \|p_i(0) - p_j(0)\| < D, j = 1, \ldots, N, j \neq i\}
\]

where \( D > 0 \) is a constant satisfying \( D > 2\ell \), and \( \|\cdot\| \) is the Euclidean norm. We suppose that for a follower, interconnection with the leader is unidirectional, that is, to say, the information of the leader may be obtained by the follower if the follower has a leader neighbor. Meanwhile, we suppose that the interconnection between any two followers is bidirectional.

The interaction network \( G(t) \) is a dynamic directed graph consisting of a vertex set \( V = \{1, \ldots, N\} \) indexed by agents and a time-varying edge set \( \mathcal{E}(t) = \{(i, j) | (i, j) \in F \times V, j \in N_i(t)\} \). Therein, the followers’ interaction network \( \tilde{G}(t) \) with vertex set \( F \) and edge set \( \mathcal{E}(t) = \{(i, j) | (i, j) \in F \times F, j \in N_i(t)\} \) is an undirected graph. In order to clarify the neighbor relationship, we introduce the adjacency matrix \( A_i(t) \) of the graph \( G(t) \) and the Laplacian matrix \( L_{N,\mathcal{E}}(t) \) of the graph \( \tilde{G}(t) \). \( A_i(t) = [w_{ij}(t)]_{N \times N} \) is defined as

\[
w_{ij}(t) = \begin{cases} 
1, & i = j = 1 \\
1, & i \in F, j \in N_i(t) \\
0, & \text{otherwise}
\end{cases}
\]

and \( L_{N,\mathcal{E}}(t) = [l_{ij}(t)]_{N \times N} \) is given by

\[
l_{ij}(t) = \begin{cases} 
-w_{ij}(t), & i \neq j \\
\sum_{k \neq i} w_{ik}(t), & i = j.
\end{cases}
\]

As \( \tilde{G}(t) \) is an undirected graph, the Laplacian matrix \( L_{N,\mathcal{E}}(t) \) is symmetric and positive semi-definite.

Given that \( G(0) \) is a leader-follower connected graph, \( G(t) \) is a fixed graph in each nonempty, bounded, and contiguous time-interval \([t_r, t_{r+1})\), where \( r = 0, 1, \ldots \).

3. Control Algorithm Design

For the external input of the leader is zero, that is,

\[
u_i(t) = \begin{bmatrix} a_i(t) \\ b_i(t) \\ c_i(t) \end{bmatrix} = 0.
\]
provided that the leader does a curve motion with a constant thrusting speed \( u_1 \), a constant rotational speed \( \omega_1 \) \(( \omega_1 \neq 0 \) ), and a constant pitch angle \( \phi_1 \) \(( \phi_1 \neq 0 \) ). Note that \( q_i = [x_i, y_i, \phi_i] \) is a constant vector. Here, \( \theta \) denotes the zero vector.

Unless specified otherwise, all variables in this paper are time-variant. Thus, in the following sections, we may use \( \dot{p}_i \) instead of \( p_i(t) \) for example. Let
\[
\dot{u}_i = u_i - w_i\hat{\omega}_i, \quad \dot{\theta}_i = \theta_i - w_i\hat{\omega}_i, \quad \dot{\phi}_i = \phi_i - w_i\hat{\omega}_i, \quad \text{and} \quad \dot{q}_i = q_i - w_i t q_i,
\]
invariant principle and graph theory are used to prove that the agents (1) applying the control protocol (9) will be asymptotically stable.

Consider a system of \( N \) agents with kinematics (1). The leader and followers are respectively steered by control protocols (7) and (9). Suppose that the initial interaction topology \( G(0) \) of the system is a leader-follower connected graph. Then the following statements hold:
- No collision happens between neighbors for two authors: D. Ruan, T. Li.
- The connectivity of the interaction topology is preserved at all times.
- The thrusting speed, the rotational speed, the yaw angle, and the pitch angle of each follower asymptotically become the same as those of the leader.
- The system approaches a desired geometric configuration \( \chi \) that minimizes the total potential.

4. Simulation Results
The initial interaction network of the leader-follower multi-agent system is a leader-follower connected graph. The initial attitudes of six agents are generate randomly. The simulation results are shown in Fig. 1.

\[
V = \sum_{i=1}^{N} \sum_{j \in N_i} V(||\hat{p}_j||) + \sum_{j \in N_i} V(||\hat{p}_j||)
\]
(10)
where \( N_i = \{ i | w_i = 1, i \in \Gamma \} \) denotes the set of followers who have one leader neighbor. The potential function \( V(||p_i(t)||) \) reaches its minimal value at the point of \( ||p_i(t)|| = 1 \).

So far, we are able to state our main result on the leader-follower formation flocking problem. LaSalle-Krasovskii invariance principle and graph theory are used to prove that the agents (1) applying the control protocol (9) will be asymptotically stable.

Definition 1 (Potential Function): Potential \( V(||\hat{p}_j||) \) is a differentiable, nonnegative, radially unbounded function of the distance \( ||\hat{p}_j|| \) between agent \( i \) and \( j \), such that
1. \( V(||\hat{p}_j||) \to \infty \) as \( ||\hat{p}_j|| \to 2R^* \) \(( R < R^* < D) \);
2. \( V(||\hat{p}_j||) \to \infty \) as \( ||\hat{p}_j|| \to 2D \);
3. \( V(||\hat{p}_j||) \) attains its unique minimum when \( ||\hat{p}_j|| \) equals to a desired value between \( 2R^* \) and \( 2D \).

Besides, the total potential of the system is
5. Conclusion

We have investigated the three-dimensional formation flocking coordination problem of multi-agent system consisting of one leader and several followers. Each agent is modeled as an extended second-order unicycle with limited communication capability. According to LaSalle-Krasovskii invariance principle and the graph theory, the followers asymptotically track the leader's velocity and form a table geometrical configuration with their neighbors. The numerical simulations are carried on six agents to further validate the functionality of the proposed algorithm for multi-agent system.

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References


