Dynamic Model and Finite-Time SMC and Backstepping Control of a Mobile-Manipulator System

Seong-Ik Han
Dep. of Electronic Eng., Pusan National University, Jangjeon-dong, Geumjeong-gu
Busan, 609-735, Korea Republic
Hyun-Uk Ha
Dep. of Electronic Eng., Pusan National University, Jangjeon-dong, Geumjeong-gu
Busan, 609-735, Korea Republic
Jang-Myung Lee
Dep. of Electronic Eng., Pusan National University, Jangjeon-dong, Geumjeong-gu
Busan, 609-735, Korea Republic
E-mail: skhan@pusan.ac.kr, hyunuk.ha@gmail.com, jmlee@pusan.ac.kr
www.pusan.ac.kr

Abstract

A mobile manipulator was designed by combining three-wheeled mobile robot equipped with the DC motor and three-links manipulator equipped with dynamixel motor. The kinematic relation and dynamic model were built via nonholonomic constraint and Euler-Lagrange equation. For the decoupled model of this system, adaptive finite-time controllers sliding mode controller (SMC) and backstepping controller were designed respectively to obtain fast tracking response. Simulation and experimental results show the efficacy of the proposed control scheme.

Keywords: Mobile-manipulator, Finite-time sliding mode control, Finite-time backsteppping control.

1. Introduction

The mobile-manipulator system has more freedom for robot works and then it has drawn more attention recently. However, it's modeling is difficult due to nonholonomic constraint of mobile platform and coupling between mobile platform and manipulator. The kinematic and dynamic coupled model for three-wheeled mobile robot and three-link manipulator system is derived using nonholomonic constraint and Euler-Lagrange equation. SMC [1],[2] and backstepping control [3] are frequently applied to control robot system but these controllers are derived based on the infinite-time stability theorem. Therefore, the convergence time is generally slow and fast response is not guaranteed. Finite-time control term [4],[5] is inserted in both controls to improve convergence time of the mobile robot and manipulator. In addition, the system parameter and uncertainty are obtained by estimation for them via adaptive observers. This leads to complex structure of the whole control system. An assumed parameter feedforward compensator is introduced to compensating unknown parameters and uncertainty.

Simulation for the decoupled mobile platform and manipulator was carried out to show the efficacy of the proposed control scheme.

2.1Description of the Mobile-Manipulator

In this section, the dynamic equations of a three-wheeled mobile manipulator system are derived using the Euler-Lagrange equation. The derived dynamics are modified from the relationship of forces acting on the body and links, and constraints between the wheel and contact surface without considering the Lagrange multiplier method, which is used to solve the nonholonomic constraint problems of mobile robots. The two-wheeled mobile manipulator is shown in Fig. 1,

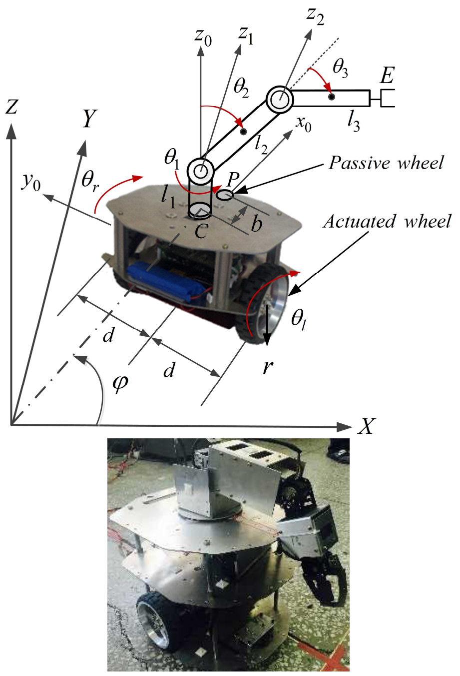

Fig. 1 Schematic diagram and photograph of the three-link and three-wheeled mobile manipulator

where variables are defined as follows: τ_r, τ_l are the torques of two wheels; τ_1, τ_2, τ_3 are the torques of the joint1, 2 and 3; θ_r, θ_l are the rotation angle of the left and right wheel of the mobile platform, respectively; v , φ are the forward velocity and the rotation angle of the mobile platform, respectively; θ_1 is the rotation angle of the link 1 with respect to z_0 axis; θ_2, θ_3 are the rotational angle of the link1 and link 2 with respect to z_1 and z_2 axis, respectively; $m_p = 5kg, m_w = 0.58kg, m_1 = 0.5kg, m_2, m_3$ are the masses of the mobile platform, wheel, link 1, link 2, and link 3, respectively; $I_p, I_{z1}, I_{z2}, I_{z3}$ are the moment of inertia of the mobile platform, link 1, link 2, and link 3, respectively; I_w is the moment of inertia of each wheel; $d = 0.145m$ the distance between the point P and wheels; $R = 0.075m$ is the radius of the wheels; $l_1 = 0.1m, l_2 = 0.2m, l_3 = 0.1m$ are the lengths of the link 1, link 2, and link 3; r_1, r_2, r_3 are the distance between joints and the center of mass of links.

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For expression simplicity, abbreviations for $s\theta = \sin\theta$, $c\theta = \cos\theta$, and $\theta_{12} = \theta_1 + \theta_2$ are introduced. By selecting the generalized coordinates are selected as $q = [q_v \ q_m]^T = [x \ y \ \varphi \ \theta_1 \ \theta_2 \ \theta_3]^T$, where $q_v = [x \ y \ \varphi]^T$ and $q_m = [\theta_1 \ \theta_2 \ \theta_3]^T$. Total kinematic energy can be expressed as:

$$
\begin{aligned}
T &= \frac{1}{2}(m_0 + m_1)(\dot{x}^2 + \dot{y}^2) + \frac{1}{2} I_o \dot{\varphi}^2 + \frac{1}{2} I_{z1}\left(\dot{\varphi} + \dot{\theta}_1\right)^2 \\
&+ \frac{1}{2} m_2 [\dot{x} - r_2 \dot{\theta}_2 s\theta_2 c_{\varphi\theta_1} - r_2(\dot{\varphi} + \dot{\theta}_1) c\theta_2 s_{\varphi\theta_1}]^2 \\
&+ \frac{1}{2} m_2 [\dot{y} - r_2 \dot{\theta}_2 s\theta_2 s_{\varphi\theta_1} + r_2(\dot{\varphi} + \dot{\theta}_1) c\theta_2 c_{\varphi\theta_1}]^2 \\
&+ \frac{1}{2} I_{z2}[(\dot{\varphi} + \dot{\theta}_1)^2 + \dot{\theta}_2^2] + \frac{1}{2} m_3 [\dot{x} - l_2 \dot{\theta}_2 s\theta_2 c_{\varphi\theta_1} \\
&- l_2(\dot{\varphi} + \dot{\theta}_1) c\theta_2 s_{\varphi\theta_1} - r_3(\dot{\theta}_2 + \dot{\theta}_3) s\theta_{23} c_{\varphi\theta_1} \\
&- r_3(\dot{\varphi} + \dot{\theta}_1) c\theta_{23} s_{\varphi\theta_1}]^2 + \frac{1}{2} m_3 [\dot{y} - l_2 \dot{\theta}_2 s\theta_2 s_{\varphi\theta_1} \\
&+ l_2(\dot{\varphi} + \dot{\theta}_1) c\theta_2 c_{\varphi\theta_1} - r_3(\dot{\theta}_2 + \dot{\theta}_3) s\theta_{23} s_{\varphi\theta_1} \\
&+ r_3(\dot{\varphi} + \dot{\theta}_1) c\theta_{23} c_{\varphi\theta_1}]^2 \\
&+ \frac{1}{2} I_{z3}[(\dot{\varphi} + \dot{\theta}_1)^2 + (\dot{\theta}_2 + \dot{\theta}_3)^2]. \qquad (1)
\end{aligned}
$$

The potential energy is obtained as follows:

$$V = m_2 g r_2 \sin\theta_2 + m_3 g \left[l_2 \sin\theta_2 + r_3 \sin(\theta_2 + \theta_3)\right]. \quad (2)$$

Using the Lagrange-Euler equation, the matrix form for the dynamic equations is written as

$$M(q)\ddot{q} + C(q,\dot{q})\dot{q} + G(q) + \tau_d = B\tau - A^T \lambda, \qquad (3)$$

where

$$M(q) = \begin{bmatrix} M_v & M_{vm} \\ M_{mv} & M_m \end{bmatrix}, \quad M_v = \begin{bmatrix} M_{xx} & 0 & M_{x\varphi} \\ 0 & M_{yy} & M_{y\varphi} \\ M_{\varphi x} & M_{\varphi y} & M_{\varphi\varphi} \end{bmatrix},$$

$$M_{vm} = \begin{bmatrix} M_{x\theta_1} & M_{x\theta_2} & M_{x\theta_3} \\ M_{y\theta_1} & M_{y\theta_2} & M_{y\theta_3} \\ M_{\varphi\theta_1} & M_{\varphi\theta_2} & M_{\varphi\theta_3} \end{bmatrix}, \quad M_{mv} = \begin{bmatrix} M_{\theta_1 x} & M_{\theta_1 y} & M_{\theta_1\varphi} \\ M_{\theta_2 x} & M_{\theta_2 y} & M_{\theta_2\varphi} \\ M_{\theta_3 x} & M_{\theta_3 y} & M_{\theta_3\varphi} \end{bmatrix}$$

$$M_m = \begin{bmatrix} M_{\theta_1\theta_1} & 0 & 0 \\ 0 & M_{\theta_2\theta_2} & M_{\theta_2\theta_3} \\ 0 & M_{\theta_3\theta_2} & M_{\theta_3\theta_3} \end{bmatrix}, \quad C(q,\dot{q}) = \begin{bmatrix} C_v & C_{vm} \\ C_{mv} & C_m \end{bmatrix},$$

$$G(q) = [0 \ 0 \ 0 \ 0 \ G_{\theta_2} \ G_{\theta_3}]^T = [G_v \ G_m]^T,$$

$\tau = [\tau_r\ \tau_l\ \tau_1\ \tau_2\ \tau_3]^T$, $A^T = \begin{bmatrix} A_v^T & 0 \\ 0 & 0 \end{bmatrix}$, $\lambda = \begin{bmatrix} \lambda_v \\ \lambda_m \end{bmatrix}$, and $B = \begin{bmatrix} B_v & 0_{3\times3} \\ 0_{2\times3} & B_m \end{bmatrix}$. (3) can be rewritten as

$$\begin{bmatrix} M_v & M_{vm} \\ M_{mv} & M_m \end{bmatrix}\begin{bmatrix} \ddot{q}_v \\ \ddot{q}_m \end{bmatrix} + \begin{bmatrix} C_v & C_{vm} \\ C_{mv} & C_m \end{bmatrix}\begin{bmatrix} \dot{q}_v \\ \dot{q}_m \end{bmatrix} + \begin{bmatrix} G_v \\ G_m \end{bmatrix} + \begin{bmatrix} \tau_{dv} \\ \tau_{dm} \end{bmatrix} = \begin{bmatrix} B_v & 0 \\ 0 & B_m \end{bmatrix}\begin{bmatrix} \tau_v \\ \tau_m \end{bmatrix} - \begin{bmatrix} A_v^T & 0 \\ 0 & 0 \end{bmatrix}\begin{bmatrix} \lambda_v \\ \lambda_m \end{bmatrix}, \quad (4)$$

where $M(q)$ is a symmetric and positive definite inertia matrix; $C(q,\dot{q})$ is a matrix of velocity-dependent centripetal and Coriolis forces; $G(q)$ is a gravitational vector; τ_d is a bounded unknown disturbance including unmodelled dynamics and exogenous disturbance; B is the input transformation matrix; and τ is an input torque vector.

Property 1. The inertia matrices M are symmetric, positive definite, and bounded. The norms of C are also bounded.

Property 2. The matrices $\dot{M} - 2C$ are skew-symmetric because of the suitable definition of the corresponding inertia and Coriolis matrix.

Therefore, this modeling method goes through the complex transformation calculation inevitably to remove the Lagrange multiplier. The resulting dynamic equations become complicated as the DOF of the attached manipulator increases.

2.2 *Kinematics of the mobile robot platform*

The nonholonomic constraint for the mobile robot is that the robot can only move in the direction normal to the axis of the driving wheels, i.e., the mobile drives under the condition of pure rolling without slipping. Therefore, the three constraints can be expressed as:

$$\dot{y}\cos\varphi - \dot{x}\sin\varphi = 0, \quad (5)$$

By selecting $q_v = [x\ y\ \varphi]^T$ as the generalized coordinates of the mobile platform, the constraint can be expressed as follows:

$$A_v(q_v)\dot{q}_v = 0, \quad (6)$$

where

$$A_v(q_v) = [-\sin\varphi\ \ \cos\varphi\ \ 0]. \quad (7)$$

The matrix $J(q_v)$ is taken as the basis for the null space of $A(q)$, $J_v^T(q_v)A_v^T(q_v) = 0$, and $J_v(q)$ can be expressed as:

$$J_v(q_v) = \begin{bmatrix} c\varphi & 0 \\ s\varphi & 0 \\ 0 & 1 \end{bmatrix}. \quad (8)$$

A reference to the mobile platform generates a trajectory for the actual platform to follow:

$$\dot{q}_{vr} = J_v(q_v)\chi_{vr}, \quad (9)$$

where $q_{vr} = [x_r\ y_r\ \varphi_r]^T$ denotes the desired time-varying position, orientation trajectory and $\chi_{vr} = [v_r\ \omega_r]^T$ denotes the reference time- varying linear and angular velocity. It is necessary to find the appropriate velocity control law $\xi_{vc} = [v_c\ \ \omega_c]^T$, such that $q_v \to q_{vr}$ as $t \to \infty$. The trajectory tracking problem is to track a reference mobile robot with a posture $q_{vr} = [x_r\ y_r\ \varphi_r]^T$. Therefore, we define the tracking error between the actual and desired posture as:

$$\tilde{q}_v = q_{vr} - q_v = \begin{bmatrix} x_r - x \\ y_r - y \\ \varphi_r - \varphi \end{bmatrix}. \quad (10)$$

The posture tracking error can be expressed as:

$$q_{ve} = \begin{bmatrix} e_x \\ e_x \\ e_\theta \end{bmatrix} = \begin{bmatrix} c\varphi & s\varphi & 0 \\ -s\varphi & c\varphi & 0 \\ 0 & 0 & 1 \end{bmatrix}\tilde{q}_v, \quad (11)$$

where e_x, e_y, and e_φ denote the tangential, normal, and orientation tracking errors of the mobile platform and manipulator, respectively. The error rate can be obtained as:

$$\begin{bmatrix} \dot{e}_x \\ \dot{e}_y \\ \dot{e}_\varphi \end{bmatrix} = \begin{bmatrix} -v + \dot{\varphi} e_y + v_r \cos e_\varphi \\ -\dot{\varphi} e_x + v_r \sin e_\varphi \\ \dot{\varphi}_r - \dot{\varphi} \end{bmatrix}. \quad (12)$$

The target or command velocity is given as:

$$\xi_c = \begin{bmatrix} v_c \\ \dot{\varphi}_c \end{bmatrix} = \begin{bmatrix} v_r \cos e_\varphi + k_x e_x \\ \dot{\varphi}_r + k_y v_r e_y + k_\varphi v_r \sin e_\varphi \end{bmatrix}, \quad (13)$$

where k_x, k_y ,and k_φ are positive constants and $v_r > 0$. This is called the extended kinematic control for mobile platform with link 1. If the perfect velocity tracking is achieved as

$$\xi_{vc} = \begin{bmatrix} v_c \\ \dot{\varphi}_c \end{bmatrix} = \begin{bmatrix} v \\ \dot{\varphi} \end{bmatrix}, \quad (14)$$

the kinematic model is then asymptotically stable with respect to the reference trajectory:

$$q_{ve} = [e_x \quad e_x \quad e_x]^T \to 0 \text{ as } t \to \infty .$$

However, in the proposed model, this kinematic technique is not required and the controller structure is simplified.

3. Design of Finite-Time Controller Design and Stability Analysis

3.1 *Design of a finite-time SMC for a mobile platform*

In (4), the mobile dynamics is separated as follows:

$$M_v \ddot{q}_v + C_v \dot{q}_v + F_v = B_v \tau_v - A_v^T \lambda_v , \quad (15)$$

where $F_v = M_{vm} \ddot{q}_m + C_{vm} \dot{q}_m + \tau_{dv}$. From (28), we have $\ddot{q}_v = J_v \dot{v}_v + \dot{J}_v v_v$. Therefore, (28) can be written as

$$M_v J_v \dot{v}_v + (M_v \dot{J}_v + C_v J_v) v_v + F_v = B_v \tau_v - A^T \lambda . \quad (16)$$

Because of $J_v^T(q_v) A_v^T(q_v) = 0$, multiplying $J_v^T(q_v)$ into the left side of (16) gives

$$M_v' \dot{v}_v + C_v' v_v + F_v' = B_v' \tau_v , \quad (17)$$

where $M_v' = J_v^T M_v J_v$, $C_v' = J_v^T (M_v \dot{J}_v + C_v J_v)$, $F_v' = J_v^T F_v$, and $B_v' = J_v^T B_v$.

Assumption 1. There are constants that satisfy the following boundedness:

$$\|M_v'\| \le \rho_{vm}, \ \|C_v'\| \le \rho_{vc}, \ \|F_v'\| \le \rho_{vf} , \quad (18)$$

where ρ_{vi}, $i = m, c, f$ are positive constants.
Consider the following signal:

$$r_v = v_{vd} - \Lambda_{v1} \int_0^t sig(e_v)^{\gamma_v} d\tau , \quad (19)$$

where $sig(e_v)^{\gamma_v} = [|e_{v1}|^{\gamma_v} sign(e_{v1}), |e_\varphi|^{\gamma_v} sign(e_\varphi)]^T$ and $0 < \gamma_v < 1$ is a constant. We then obtain the following

$$\dot{r}_v = \dot{v}_{vr} - \Lambda_{v1} sig(e_v)^{\gamma_v} . \quad (20)$$

The finite-time sliding mode surface s_v is defined as

$$\begin{aligned} s_v &= v_v - r_v \\ &= v_v - v_{vr} + \Lambda_{v1} \int_0^t sig(e_v)^{\gamma_v} d\tau \\ &= e_v + \Lambda_{v1} \int_0^t sig(e_v)^{\gamma_v} d\tau . \end{aligned} \quad (21)$$

Using (19), (20), and (21), it follows that

$$\begin{aligned} M_v' \dot{s}_v &= M_v' \dot{v}_v - M_v' \dot{r}_v \\ &= -C_v' v_v - F_v' + B_v' \tau_v - M_v' \dot{r}_v \\ &= -C_v'(r_v + s_v) - F_v' + B_v' \tau_v - M_v' \dot{r}_v \\ &= -C_v' s_v - M_v' \dot{r}_v - C_v' r_v - F_v' + B_v' \tau_v . \end{aligned} \quad (22)$$

We define the Lyapunov function as follows:

$$V_v = \frac{1}{2} s_v^T M_v' s_v + \frac{1}{2}(e_x^2 + e_y^2) + \frac{1 - \cos e_\varphi}{k_y} . \quad (23)$$

Considering (24), (39), and property 2, the time derivative of (40) becomes

$$\begin{aligned} \dot{V}_v &= s_v^T M_v' \dot{s}_v + \frac{1}{2} s_v^T \dot{M}_v' s_v + e_x \dot{e}_x + e_y \dot{e}_y + \frac{\dot{e}_\varphi \sin e_\varphi}{k_y} \\ &= s_v^T [-M_v' \dot{r}_v - C_v' r_v - F_v' + B_v' \tau_v] - k_1 e_x^2 \end{aligned}$$

$$-\frac{k_3 v_r \sin^2 e_\varphi}{k_2}$$
$$\le s_v^T \left(\rho_{vm}\dot{r}_v + \rho_{vc} r_v + \rho_{vf} + B_v' \tau_v\right)$$
$$-k_1 e_x^2 - \frac{k_3 v_r \sin^2 e_\varphi}{k_2}. \quad (24)$$

The control input is specified as

$$\tau_v = \tau_{veq} + \tau_{vr} + \tau_f, \quad (25)$$
$$\tau_{veq} = -B_v'^{-1}\left[-\Lambda_{v2} s_v - \rho_{vm}\|\dot{r}_v\| - \rho_{vc}\|r_v\| - \rho_{vf}\right], \quad (26)$$
$$\tau_{vr} = -\Lambda_{vr} s_v (\|s_v\| + \varepsilon_v)^{-1}, \quad (27)$$
$$\tau_f = -\Lambda_{vf}\, sig(s_v)^{\gamma_v}. \quad (28)$$

where $\Lambda_{vf} = diag(c_{vf}, c_{\varphi f}) > 0$ is a constant matrix. Substituting (25) into (24), we have

$$\dot{V}_v \le -\Lambda_{v2} s_v^T s_v - \Lambda_{vr} s_v^T s_v (\|s_v\| + \varepsilon_v)^{-1} - \sum_{i=1}^{2} c_i |s_{vi}|^{\gamma_v+1}$$
$$-k_1 e_x^2 - \frac{k_3 v_r \sin^2 e_\varphi}{k_2}$$
$$\le -\Lambda_{v2} s_v^T s_v - \sum_{i=1}^{2} c_i |s_i|^{\gamma_v+1}$$
$$\le -\lambda_{\min}(\Lambda_{v2}) s_v^T s_v - \lambda_{\min}(c_i) \sum_{i=1}^{2} (|s_{vi}|^2)^{(\gamma_v+1)/2}$$
$$\le -\frac{2\lambda_{\min}(\Lambda_{v2})}{\lambda_{\max}(\Gamma_v)} V_{v3} - \frac{2^{(\gamma_v+1)/2}\lambda_{\min}(c_i)}{\lambda_{\min}(c_i)} V_{v3}^{\gamma_v}$$
$$\le -k_{v1} V_{v3} - k_{v2} V_{v3}^{\gamma_v}, \quad (29)$$

where $k_{v1} = \frac{2\lambda_{\min}(\Lambda_{v2})}{\lambda_{\max}(\Gamma_v)}$, $k_{v2} = \frac{2^{(\gamma_v+1)/2}\lambda_{\min}(c_i)}{\lambda_{\min}(c_i)}$.

Lemma 1: From the definition of finite-time stability [4],[5], the equilibrium point $s = 0$ of is globally finite-time stable; i.e., for any given initial condition $s(0) = s_0$, an extended Lyapunov description of finite-time is given as the following inequality:

$$\dot{V}(s) \le -\xi_1 V(s) - \xi_2 V^\gamma(s), \quad (30)$$

where $\xi_1 > 0$, $\xi_2 > 0$, and $0 < \gamma < 1$. $V(t)$ converges to an equilibrium point in finite-time t_s given by

$$t_s \le \frac{1}{\xi_1(1-\gamma)} \ln \frac{\xi_1 V^{1-\gamma}(s_0) + \xi_2}{\xi_2}. \quad (31)$$

Therefore, using Lemma2 and (84), the finite convergence time of the mobile robot is given as

$$t_{vs} \le \frac{1}{k_{v1}(1-\gamma_v)} \ln \frac{k_{v1} V_v^{1-\gamma_v}(s_{v0}) + k_{v2}}{k_{v2}}. \quad (32)$$

3.2. *Design of a finite-time backstepping controller for a mobile platform*

The state space model of the manipulator from (4) can be expressed as follows:

$$\dot{x}_3 = x_4,$$
$$\dot{x}_4 = M_m^{-1}\left(-C_m \dot{q}_m - G_m - F_{mv} + B_m \tau_m\right),$$
$$y_m = x_3, \quad (33)$$

where $x_3 = [\theta_1, \theta_2, \theta_3]^T$, $x_4 = \dot{x}_3 = [\dot{\theta}_1, \dot{\theta}_2, \dot{\theta}_3]^T$, and $F_{mv} = M_{mv}\ddot{q}_v + C_{mv}\dot{q}_v + \tau_{dm}$.

Assumption 2. There are constants that satisfy the following boundedness:

$$\|M_m\| \le \rho_{mm}, \ \|C_m\| \le \rho_{mc}, \ \|G_m\| \le \rho_{mg}, \ \|F_m\| \le \rho_{mf}, \quad (34)$$

where ρ_{mi}, $i = m, c, g, f$ are positive constants.

The tracking error and x_4 are written as

$$z_3 = x_3 - y_{mr}, \quad (35)$$
$$z_4 = x_4 - \alpha_3. \quad (36)$$

The following Lyapunov function is defined:

$$V_{m1} = \frac{1}{2} z_3^T z_3. \quad (37)$$

Using (35) and (36), we then have

$$\dot{V}_{m1} = z_3^T \dot{z}_3 = z_3^T (z_4 + \alpha_3 - \dot{y}_{mr}). \quad (38)$$

We specify the finite-time virtual control as follows:

$$\alpha_3 = -c_3 z_3 - \zeta_3 sig(z_3)^{\gamma_3} + \dot{y}_{mr}, \quad (39)$$

where $c_3 = diag(c_{\theta_1}, c_{\theta_2}, c_{\theta_3}) > 0$ is a constant matrix, $\zeta_3 = diag(\zeta_{\theta_1}, \zeta_{\theta_2}, \zeta_{\theta_3}) > 0$ is a constant matrix, $sig(z_3)^{\gamma_3} = \left[|z_{3\theta_1}|^{\gamma_{3\theta_1}} \operatorname{sgn}(z_{3\theta_1}), |z_{3\theta_2}|^{\gamma_{3\theta_2}} \operatorname{sgn}(z_{3\theta_2}), |z_{3\theta_3}|^{\gamma_{3\theta_3}} \operatorname{sgn}(z_{3\theta_3}) \right]$ and $0 < \gamma_3 < 1$ is a constant.

Therefore, we have

$$\dot{V}_{m1} = -c_3 z_3^T z_3 - \zeta_3 |z_3|^{\gamma_3+1} + z_3^T z_4. \quad (40)$$

We redefine the Lyapunov function as follows:

$$V_{m3} = V_{m1} + \frac{1}{2} z_4^T M_m z_4. \quad (41)$$

We obtain

$$\dot{V}_{m3} = \dot{V}_{m1} + z_4^T M_m \dot{z}_4 + \frac{1}{2} z_4^T \dot{M}_m z_4 = -c_3 z_3^T z_3 + z_4^T \left(\rho_{mc} \|x_4\| + \rho_{mg} + \rho_{mf} + B_m \tau_m + \rho_{mm} \|\dot{\alpha}_3\| + z_3 \right). \quad (42)$$

Selecting the finite-time control input and adaptive laws as follows:

$$\tau_m = B_m^{-1} \left(-c_3 z_4 - z_3 - \rho_{mm} \|\dot{\alpha}_3\| - \rho_{mc} \|x_4\| - \rho_{mg} - \rho_{mf} - \zeta_4 sig(z_4)^{\gamma_4} \right), \quad (43)$$

Then, we obtain

$$\dot{V}_{m3} \le -\sum_{i=3}^{4} c_i z_i^T z_i - \sum_{i=3}^{4} \zeta_i |z_i|^{\gamma_i+1} \le -k_{m1} V_{m3} - k_{m2} V_{m3}^{\gamma_m}, \quad (44)$$

where $k_{m1} = \dfrac{2\lambda_{\min}(c_m)}{\lambda_{\max}(\Gamma_m)}$, $k_{m2} = \dfrac{2^{\gamma_m} \lambda_{\min}(\zeta_i)}{\lambda_{\min}(\zeta_i)}$, and $\gamma_m = \min[(\gamma_i + 1)/2], i = 3, 4$. Therefore, using Lemma1, the finite convergence time of the manipulator control system is given as

$$t_{ms} \le \frac{1}{k_{m1}(1-\gamma_m)} \ln \frac{k_{m1} V_{m3}^{1-\gamma_m}(z_{m0}) + k_{m2}}{k_{m2}}.$$

4. Simulation Example

To validate the proposed control scheme, the infinite-time backstepping controller (BSC) and finite-time backstepping controller (FBSC) with infinite-time SMC (SMC) and finite-time SMC(FSMC) were designed. Simulation results of each control system for the mobile manipulator system are described in Figs. 2 and 3, where the proposed finite-time control reveals more fast convergence performance than the infinite-time based control sytems.

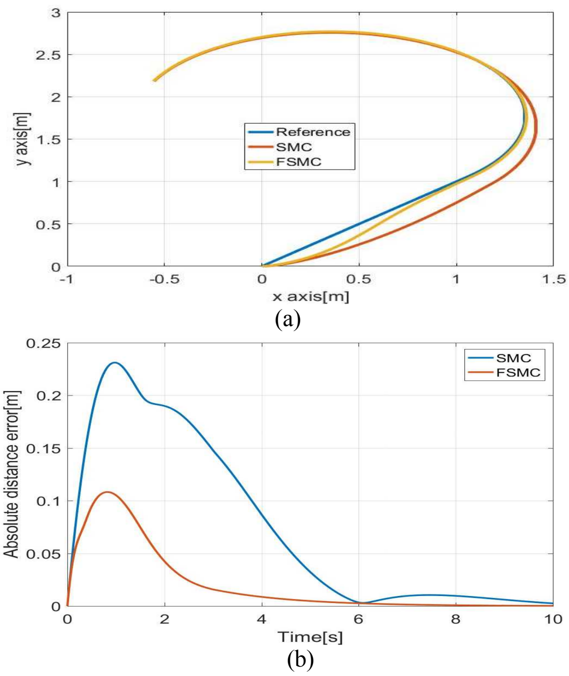
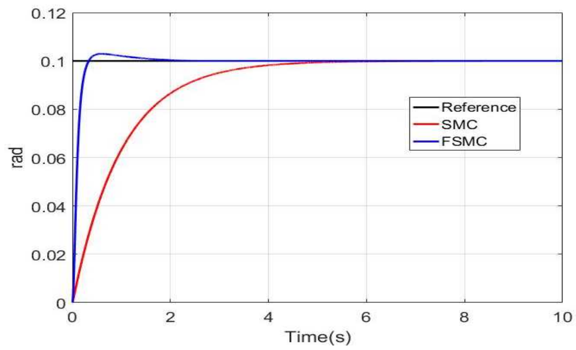
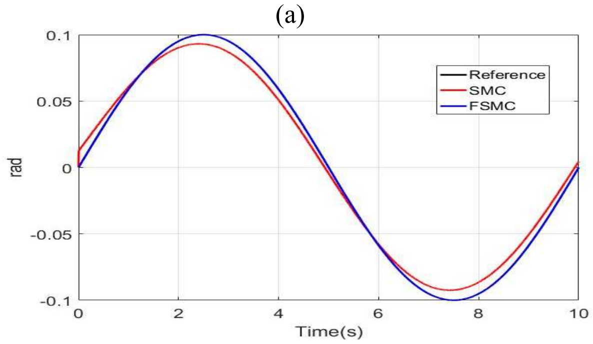


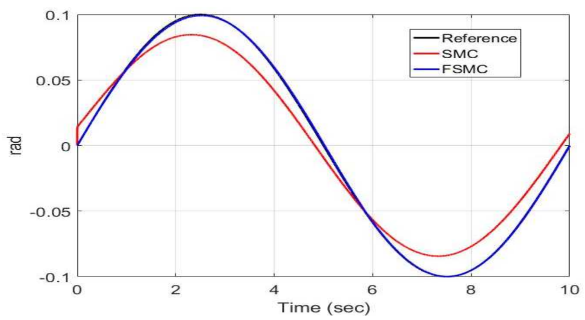
Fig. 2. Simulation results of the mobile platform. (a) Tracking outputs. (b) Tracking errors.



(a)



(b)



5. Conclusion

Finite-time SMC and backstepping control schemes to guarantee the fast error convergence and small tracking error performance for a mobile-manipulator system are proposed. A finite sliding mode surface and a virtual finite-time error surface are defined to obtain finite-time performance. The finite-time convergence is proved by the finite-time stability analysis of Lyapunov function. Simulation for a mobile manipulator system confirms the theoretical proposal.

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