Modeling and Control of a Quadrotor Vehicle Subject to Disturbance Load

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Abstract

In this paper, a dynamic model of quadrotor vehicle is derived for theoretic and practical evaluation. Four transfer functions in different channels are converted from the state equations. To study the behavior of quadrotor subject to the external disturbance load, the FLC (fuzzy logic controller) is designed to compare with the PID (proportional-integral-derivative) controller. Subsequently, Liapounov function is applied for stability analysis. Finally, simulation results are presented to illustrate the performance between FLC and PID. Considering model error, the evaluation simulations are divided into two parts, which describe the ability for rejecting external disturbance, setpoint tracking and disturbance rejection respectively. The simulation scheme demonstrates the FLC method outperforms the PID control scheme.

Keywords: Disturbance load; Dynamic model; Fuzzy logic controller; PID; Quadrotor.

1. Introduction

In the recent years, quadrotor vehicle has been paid special attention due to its small size and flexible maneuverability1. The previously developments like PID control, Linear Quadratic control, back-stepping, slide mode control2-3 are still used nowadays as suitable solutions for the quadrotors autonomous flying1,4. Considering the robust control in terms of process modeling errors and disturbance load, Lyapunov theory is introduced for the flight control design5-7. Backstepping is well known and widely used in the control of nonlinear system, especially in the trajectory tracking control of the UAV8-10.

However, the application of back-stepping depends on the actuated modeled system dynamics. Under-actuated model will dramatically decrease the control quality. In this paper, the Liapounov function and fuzzy logic controller is proposed to achieve better performance for quadrotor control.

2. Modeling of quadrotor vehicle

As shown in Fig.1, the quadrotor is an X-shaped four-rotor aircraft, with each rotor located in endpoints.

Fig. 1. Designed Quadrotor

The quadrotor drives the posture and movement depending on the rotational thrust and torque of four rotors. As shown in Fig. 2, rotors 2, 4 and 1, 3 rotate in opposite direction for eliminating the yawing torque. If the four rotors rotate at the same speed, the aircraft will produce vertical motion. Change the speeds of rotors 4, 2 and keep the speeds of rotors 1, 3, the aircraft will produce pitch movement. Similarly, roll movement result from 1, 3 rotor’s speeds change. If the yawing torque in the different diagonals cannot be cancelled, the yaw movement is achieved.
Let the vector \( p = [x, y, z]^T \) denote the relative position of a quadrotor with respect to an inertial coordinate, and \( q = [\alpha, \beta, \gamma]^T \) represents attitude angles of the quadrotor. Set \( O_M \) as the original point. \( XYZ \) is the body fixed coordinates. The transformation matrix between the two coordinates is written as:

\[
R_{xy} = \begin{bmatrix}
1 & 0 & 0 \\
0 & \cos \alpha & \sin \alpha \\
0 & -\sin \alpha & \cos \alpha
\end{bmatrix}
\]

Assume that the thrust generated by the four rotors is perpendicular to the aircraft. Therefore, in the body coordinates, the thrust is expressed as:

\[
F_M = \begin{bmatrix}
0.0, & \sum_{i=1}^{4} F_i \end{bmatrix}^T
\]

Considering equations (1), (2), in inertial coordinate, the thrust analysis is:

\[
F = \begin{bmatrix}
F_x \\
F_y \\
F_z
\end{bmatrix} = R_{xy} F_M = \sum_{i=1}^{4} F_i = \begin{bmatrix}
-\sin \beta \\
\sin \alpha \cos \beta \\
\cos \alpha \cos \beta
\end{bmatrix}
\]

Let \( m \) denote the quality of the quadrotor. The acceleration of quadrotor in the inertial coordinate is given by:

\[
\ddot{p} = \begin{bmatrix}
\ddot{x} \\
\ddot{y} \\
\ddot{z}
\end{bmatrix} = \begin{bmatrix}
F_x/m \\
F_y/m \\
(F_z - mg)/m
\end{bmatrix}
\]

Similarly, the angular acceleration can be written as:

\[
\dot{q} = \begin{bmatrix}
\dot{\alpha} \\
\dot{\beta} \\
\dot{\gamma}
\end{bmatrix} = \begin{bmatrix}
(r(F_x - F_z)/I_x \\
(r(F_y - F_z)/I_y \\
(M_1 - M_2 + M_3 - M_4)/I_z
\end{bmatrix}
\]

Where, \( r \) is the length from rotor to the center of the mass of the quadrotor. \( F_i \) denotes the thrust of \( i \)th rotor. \( I_x, I_y, \) and \( I_z \) are the rotational inertia of the quadrotor around \( X, Y \) and \( Z \)-axis respectively. \( M_i \) is torque generated by \( i \)th rotor.

In terms of the equations (4), (5), the dynamic equations can be yielded:

\[
T = \begin{bmatrix}
\dot{x} \\
\dot{y} \\
\dot{z}
\end{bmatrix} = \begin{bmatrix}
\ddot{F_x} \\
\ddot{F_y} \\
\ddot{F_z}
\end{bmatrix}
\]

Similarly, the angular acceleration can be written as:

\[
\dot{p} = \begin{bmatrix}
\dot{x} \\
\dot{y} \\
\dot{z}
\end{bmatrix} = \begin{bmatrix}
\ddot{F_x}/m \\
\ddot{F_y}/m \\
(F_z - mg)/m
\end{bmatrix}
\]

The transfer function of quadrotor can be presented:

\[
G(s) = C(sI - A)^{-1}B + D = \begin{bmatrix}
0 & 0 & 0 \\
0 & 1/I_1 & 0 \\
0 & 0 & 1/I_3 + 1/I_2
\end{bmatrix}
\]

The parameters of the designed quadrotor are measured as follows:

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>( I_x )</td>
<td>0.1</td>
</tr>
<tr>
<td>( I_y )</td>
<td>0.1</td>
</tr>
<tr>
<td>( I_z )</td>
<td>0.1</td>
</tr>
<tr>
<td>( m )</td>
<td>0.1</td>
</tr>
<tr>
<td>( g )</td>
<td>9.8</td>
</tr>
</tbody>
</table>

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### 3. Control design and stability analysis

#### 3.1. PID control method

The PID control with simple structure has excellent stability by selecting appropriate proportion, differentiation and integration coefficients for disturbance rejection. Taking pitch channel as an example, the simulation results are conducted on the block diagram of PID control system in Fig. 3.

![Fig. 3. Block diagram of PID control system](image)

**Fig. 3.** Block diagram of PID control system

#### 3.2. Fuzzy logic modified PID control implementation

The fuzzy logic modified PID controller (FLC) is also proposed in the same manner to compare with the performance of PID controller. A block diagram of fuzzy controller structure with a disturbance load is shown in Fig. 4.

![Fig. 4. Block diagram of FLC control system](image)

**Fig. 4.** Block diagram of FLC control system

The error of angle (expressed by $e$) and change of the angle error (expressed by $\dot{e}$) are regard as inputs of FLC; $U$ is the control output of the FLC. As a rule of a thumb, $e$ and $\dot{e}$ can be divided into seven grades (As shown in Fig. 5.).

<table>
<thead>
<tr>
<th>Item</th>
<th>Values</th>
</tr>
</thead>
<tbody>
<tr>
<td>$r$ (m)</td>
<td>0.245</td>
</tr>
<tr>
<td>$m$ (kg)</td>
<td>1.05</td>
</tr>
<tr>
<td>$I_x \times 10^3$/(kg.m²)</td>
<td>6.2</td>
</tr>
<tr>
<td>$I_y \times 10^3$/(kg.m²)</td>
<td>6.2</td>
</tr>
<tr>
<td>$L \times 10^3$/(kg.m²)</td>
<td>3</td>
</tr>
</tbody>
</table>

#### 3.3. Convergence analysis

To further illustrate quadrotor’s stability, a Lyapunov function approval is introduced. In terms of the pitch motion, it can be described as:

$$e = \alpha - \alpha_0, \quad e = \dot{e} - \dot{\alpha}_0$$

(9)

In which, $\alpha$ is the measured angle, $\alpha_0$ is the desired angle, we usually set $\alpha_0 = 0$. It is obtained:

$$\dot{x}_1 = x_2 = \dot{\alpha} - \dot{\alpha}_0$$

$$\dot{x}_2 = \ddot{\alpha} - \ddot{\alpha}_0 = \frac{u_2}{I_x} - \ddot{\alpha}_0$$

(10)

Assume nonlinear systems:

$$\ddot{x} = f(x) = [x_1, x_2]^T = [x_2, \dot{x}_2]^T$$

(11)

is asymptotically stable on the equilibrium $x_c = [0, 0]$. The progress of looking for a Lyapunov function is organized as follows:

1. Assume $V(x)$ is a scalar function of $x$, the gradient of $V(x)$ is:

$$\nabla V = \begin{pmatrix} a_{11} x_1 + a_{12} x_2 \\ a_{21} x_1 + a_{22} x_2 \end{pmatrix} = \begin{pmatrix} \nabla V_1 \\ \nabla V_2 \end{pmatrix}$$

(12)

Where, $a_{11}$, $a_{12}$, $a_{21}$, $a_{22}$ are unknown coefficients.

2. Then we have:

$$\dot{V}(x) = (\nabla V)^T \ddot{x} = (a_{11} x_1 + a_{12} x_2, a_{21} x_1 + a_{22} x_2, \dot{x}_1, \dot{x}_2)^T$$

(13)

3. Try to choose $a_{11} = a_{22} = 1, a_{12} = a_{21} = 0$, we get

$$\dot{V}(x) = x_1 \dot{x}_1 + x_2 \dot{x}_2$$

(14)

Recall (12), we have
\[
VV = \begin{bmatrix} x_1 \\ x_2 \end{bmatrix}
\]

(15)

It is obvious to meet curl equation:

\[
\frac{\partial V}{\partial x_2} = \frac{\partial V}{\partial x_1}, \quad \text{namely,} \quad \frac{\partial x_1}{\partial x_2} = \frac{\partial x_2}{\partial x_1} = 0
\]

(16)

which indicates the parameters chosen above is rational.

Therefore, the Lyapunov function can be yield as follows:

\[
V(x) = \int (\nabla V)^T \nabla x = \int x_1 dx_1 + \int x_2 dx_2
\]

\[
= \frac{1}{2}(x_1^2 + x_2^2)
\]

(17)

Invoking (10), (14), we have

\[
V(x) = x_1\dot{x}_1 + x_2\dot{x}_2 = (\ddot{x} - \ddot{\alpha}_0)(\alpha + \ddot{\alpha}_0 - \dot{\alpha}_0) = (\ddot{x} - \ddot{\alpha}_0)(\alpha - \dot{\alpha}_0 - \ddot{\alpha}_0 + u_2/I_x)
\]

(18)

To achieve the stability of the system, the condition \(\dot{V}(x) \leq 0\) is necessary. Due to, \(\alpha_0 = 0\) we have \(\dot{\alpha}_0 = \ddot{\alpha}_0 = 0\). Therefore, to achieve \(V(x) \leq 0\), we just need design a fuzzy controller to adjust \(\alpha, \dot{\alpha}\) for satisfying either of the following conditions:

\[
\dot{\alpha} > 0 \quad \text{and} \quad \alpha + u_2/I_x < 0
\]

(19)

\[
\dot{\alpha} < 0 \quad \text{and} \quad \alpha + u_2/I_x > 0
\]

(20)

From the mentioned above, we can make a conclusion that the convergence in the model can be guaranteed.

4. Simulation

In this section, the performance of PID and FLC are compared by simulations, which are mainly divided into two parts. The first part is conducted in the presence of disturbance load; the second part presents the case with modeling error.

4.1. No modeling error

4.1.1. Disturbance rejection

According to Fig. 4, the time of simulation is 20s. During \(t=4.4s\), a pulse signal disturbance load is introduced. The simulation result is shown in Fig. 6. The PID controller responds from peak to bottom during \(t=4.4-6.59s\), the overshoot angle in negative direction reaches -0.85°. At about \(t=16.09s\), the angle reaches the desired value.

Fig. 6. Simulation results for fighting against a disturbance

For FLC, the response wave reaches peak at about \(t=4.37s\), and reaches trough at about \(t=5.41s\). The overshoot angle in negative direction is -0.8°. At about \(t=13.86s\), the controlled angle reaches 0°. The FLC has faster response speed, smaller reverse overshoot than PID. Thus, fuzzy PID control scheme has stronger ability to resist a disturbance load.

4.1.2. Setpoint tracking

To identify the tracking abilities of PID and FLC, set the disturbances to zero. According to Fig. 3, Fig. 4, a step signal is introduced at initial time.

Fig. 7. Simulation results of setpoint tracking

In Fig. 7, when a step signal is introduced, the two systems make response at the same time. With PID, the response is faster than that of FLC during 1s-2s. As for FLC, it reaches the desired value at 10.3s, which is slightly earlier than that of PID.

4.1.3. Setpoint tracking& disturbance rejection

Fig. 8 shows the simulation results of the step response with a disturbance load. When a step signal is introduced at the beginning, the response of two systems just like what the Fig. 7 shows.

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When a disturbance is introduced within $t=8$-8.4s, it can be observed that both of two controllers quickly moves towards the maximum in negative direction. However, the response of FLC is more sensitive to the disturbance than that of traditional PID controller. It is apparent that the system controlled by FLC reaches the desired angle quickly.

4.2. With modeling error

A Considering process modeling error, a third order transfer function in pitch channel is applied as follows:

$$G(s) = \frac{56.95s + 4391}{s^3 + 105s^2 + 870s + 4430} \quad (21)$$

4.2.1. Setpoint tracking & disturbance rejection

Let the gain of disturbance be zero, a step signal is given to identify the ability of setpoint tracking of the third order transfer function. Meanwhile, a disturbance is introduced during 8s-8.4s. The simulation result is presented in Fig.9.

The result shows the angle can be controlled to its desired value within 20s. And the overshoot angle of FLC in negative direction is smaller. Therefore, FLC provides better performance. All the simulation results mentioned above indicate that the FLC controller can efficiently respond to the outside disturbance, thus show better performances than its counterpart. The reason lies in the fact that the FLC can automatically adjust $k_p, k_i, k_d$ in time according to the external conditions so as to maintain the stability of the control system. However, the three parameters of traditional PID is unchanged as soon as they are set in the initial time, that is to say, the traditional PID does not have the adaptive characteristics.

5. Conclusion

In the paper, a fuzzy logic modified PID (FLC) control scheme is designed to control stability of the quadrotor. Based on theoretical analysis, a Liapounov function is constructed to prove that the stability of the control system can be achieved. The difference between the actual angle and desired angle is presented as the error. The error and change-in-error are applied as inputs of the FLC. To further test the performance of the designed controller subjecting to the disturbance load, the simulation are conducted in MATLAB/ Simulink. The simulation results demonstrate that FLC response more quickly than PID control method, moreover, in terms of FLC, the angle can be controlled to its desired value within less time compared to its counterpart.

References