Simulated Kalman Filter with Randomized Q and R Parameters

Nor Hidayati Abdul Aziz, Nor Azlina Ab. Aziz
Faculty of Engineering and Technology, Multimedia University
Bukit Beruang, Melaka, Malaysia

Zuwairie Ibrahim, Saifudin Razali, Mohd Falfazli Mat Jusof
Faculty of Electrical and Electronics Engineering, Universiti Malaysia Pahang
Pekan, Pahang, Malaysia

Khairul Hamimah Abas, Mohd Saberi Mohamad
Universiti Teknologi Malaysia
Skudai, Johor, Malaysia

Norrima Mokhtar
Faculty of Engineering, University of Malaya
50603 Kuala Lumpur, Malaysia

E-mail: hidayati.aziz@mmu.edu.my, azlina.aziz@mmu.edu.my, zuwairie@ump.edu.my, saifudin@ump.edu.my, mfalfazli@ump.edu.my, khairulhamimah@utm.my, saberi@utm.my, norrimamokhtar@um.edu.my

Abstract

Inspired by Kalman filtering, simulated Kalman filter (SKF) has been introduced as a new population-based optimization algorithm. The SKF is not a parameter-less algorithm. Three parameter values should be assigned to *P*, *Q*, and *R*, which denotes error covariance, process noise, and measurement noise, respectively. While analysis of *P* has been studied, this paper emphasizes on *Q* and *R* parameters. Instead of using constant values for *Q* and *R*, random values are used in this study. Experimental result shows that the use of randomized *Q* and *R* values did not degrade the performance of SKF and hence, one step closer to the realization of a parameter-less SKF.

Keywords: Optimization, simulated Kalman filter, random, error covariance, process noise, measurement noise.

1. Introduction

The simulated Kalman filter (SKF) [1] is a relatively new optimization algorithm compared to some well-known optimizer such as genetic algorithm (GA) [2], particle swarm optimization (PSO) [3], and gravitational search algorithm (GSA) [4]. Since the first introduction of SKF, it has been modified and applied extensively. These include fundamental improvements to SKF. For example, to solve combinatorial optimization problems, angle-modulated SKF [5], binary SKF [6], and distance evaluated SKF [7] have been presented to enable the SKF to operate in binary search space. The SKF also has been hybridized with PSO and GSA [8-9]. From the application point of view, airport scheduling has been solved using SKF [10]. Also, recently, the SKF has been employed to select the most significant features for peak detection of EEG signal [11].

2. The Original and Modified SKF Algorithms

The simulated Kalman filter (SKF) algorithm is illustrated in Figure 1. Consider n number of agents, SKF algorithm begins with initialization of n agents, in which the states of each agent are given randomly. The maximum number of iterations, t_{max}, is defined. The initial value of error covariance estimate, $P(0)$, the process noise value, Q, and the measurement noise value, R, are also defined during initialization stage. Then, every agent is subjected to fitness evaluation to produce initial solutions, $\boldsymbol{X}(0) = \{\boldsymbol{X}_1^d(0), \boldsymbol{X}_2^d(0), \ldots, \boldsymbol{X}_{n-1}^d(0), \boldsymbol{X}_n^d(0)\}$ where d refers to the dimensional of the problem. The fitness values are compared and the agent having the best fitness value at every iteration, t, is registered as $\boldsymbol{X}_{\mathbf{best}}(t)$. For function minimization problem,

$$\boldsymbol{X}_{\boldsymbol{best}}(t) = \min_{i \in 1,\ldots,n} fit_i(\boldsymbol{X}(t)) \quad (1)$$

whereas, for function maximization problem,

$$\boldsymbol{X}_{\boldsymbol{best}}(t) = \max_{i \in 1,\ldots,n} fit_i\big(\boldsymbol{X}(t)\big) \quad (2)$$

The-best-so-far solution in SKF is named as $\boldsymbol{X}_{\mathbf{true}}$. The $\boldsymbol{X}_{\mathbf{true}}$ is updated only if the $\boldsymbol{X}_{\mathbf{best}}(t)$ is better ($\boldsymbol{X}_{\mathbf{best}}(t) < \boldsymbol{X}_{\mathbf{true}}$ for minimization problem, or $\boldsymbol{X}_{\mathbf{best}}(t) > \boldsymbol{X}_{\mathbf{true}}$ for maximization problem) than the $\boldsymbol{X}_{\mathbf{true}}$.

The prediction step is subjected to the following time-update equations:

$$\boldsymbol{X}_i^d(t|t) = \boldsymbol{X}_i^d(t) \quad (3)$$

$$P(t|t) = P(t) + Q \quad (4)$$

where $\boldsymbol{X}_i^d(t)$ and $\boldsymbol{X}_i^d(t|t)$ are the current state and transition/predicted state, respectively, and $P(t)$ and $P(t|t)$ are previous error covariant estimate and transition error covariant estimate, respectively.

The next step is measurement. Measurement, $\boldsymbol{Z}_i(t)$, of each individual agent is simulated based on the following equation:

$$\boldsymbol{Z}_i^d(t) = \boldsymbol{X}_i^d(t|t) + \sin(rand \times 2\pi) \times \left|\boldsymbol{X}_i^d(t|t) - \boldsymbol{X}_{\boldsymbol{true}}\right| \quad (5)$$

where $rand$ is a uniformly distributed random number in the range of $[0,1]$.

The final step is the estimation. During this step, Kalman gain, $K(t)$, is computed as follows:

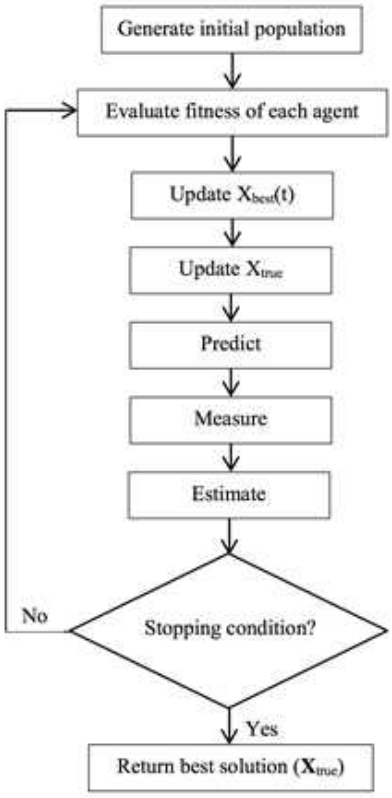

Fig. 1. The SKF algorithm.

$$K(t) = \frac{P(t|t)}{P(t|t)+R} \quad (6)$$

Then, the estimation of next state, $\boldsymbol{X}_i(t)$, is computed based on Eq. (7).

$$\boldsymbol{X}_i^d(t+1) = \boldsymbol{X}_i^d(t|t) + K(t) \times (\boldsymbol{Z}_i^d(t) - \boldsymbol{X}_i^d(t|t)) \quad (7)$$

and the error covariant is updated based on Eq. (8).

$$P(t+1) = \big(1 - K(t)\big) \times P(t|t) \quad (8)$$

Finally, the next iteration is executed until the maximum number of iterations, t_{max}, is reached.

The SKF algorithm requires the determination of P, Q, and R parameters. Earlier, P = 1000 and $Q = R$ = 0.5 has been suggested [1]. However, based on an analysis on P, it was found that the parameter P alone did not affect the performance of SKF when $Q = R$ = 0.5. Hence, Q and R parameters greatly affect the performance of the SKF. Since the determination of Q and R parameters is critical, this study suggests the determination of those parameters

Table 1. Performance comparison between the original SKF with SKF with random Q and R values.

Function No.	Original SKF	Random QR SKF	Sign
1	4.70E+06	4.70E+06	=
2	2.45E+07	3.53E+07	<
3	18148	19599	<
4	532.77	522.09	>
5	520.01	520.01	=
6	633.44	631.86	>
7	700.25	700.31	<
8	807.98	807.73	>
9	1059.1	1064	<
10	1335.2	1366.4	<
11	6249.4	6166.4	>
12	1200.2	1200.2	=
13	1300.6	1300.6	=
14	1400.3	1400.3	=
15	1551.7	1547.8	>
16	1619.1	1619.3	<
17	9.08E+05	8.80E+05	>
18	6.94E+06	3.06E+06	>
19	1950.2	1940.8	>
20	34799	33350	>
21	1.19E+06	1.10E+06	>
22	3429.1	3473.6	<
23	2645.7	2645.5	>
24	2667.2	2664.6	>
25	2730.4	2730.8	<
26	2766.4	2778.4	<
27	3883.3	3876.1	>
28	7223.4	6931.7	>
29	5997.8	9398.7	<
30	19753	19184	>

= similar > Random QR SKF is better < Original SKF is better

could be avoided. Thus, in this work, random numbers, *randn* ∈ [0,1], normally distributed at the mean of 0.5, is generated whenever Q and R values are required, at every iteration, every agent, and every dimension.

In the proposed SKF with randomized Q and R values, Eq. (2) is replaced with Eq. (9) as follows:

$$P_i^d(t|t+1) = P_i^d(t) + randn_i^d \qquad (9)$$

since the P parameter is dependent on Q. On the other hand, Eq. (6) is replaced with Eq. (10) since the Kalman gain, $K_i(t)$, is dependent on R.

$$K_i^d(t) = [P_i^d(t|t+1)]/[P_i^d(t|t+1) + randn_i^d] \qquad (10)$$

Subsequently, Eq. (8) is replaced with Eq. (11).

$$P_i^d(t+1) = (1 - K_i^d(t)) \times P_i^d(t|t+1) \qquad (11)$$

3. Experiment, Result, and Discussion

The experiment was conducted based on CEC2014 benchmark dataset for function minimization problems (http://www.ntu.edu.sg/home/EPNSugan/index_files/CEC2014/CEC2014.htm), which includes 30 set of problems. Every run consists of 10,000 function evaluations. After a single run, the same experiment was implemented again until 50 results were recorded and average values were calculated. The average value for every function is depicted in Table 1. Note that for original SKF algorithm, P = 1000 and $Q = R$ = 0.5. The

sign indicates performance comparison between two algorithms for every problem. Based on the result, however, general conclusion cannot be made. Figure 2 shows an example of convergence curve, which mostly the same.

Table 2 shows further analysis based on Wilcoxon signed-rank test. The R^+ and R^- values were calculated and those values were compared to a threshold value, which is 137. Since both R^+ and R^- values are greater than 137, there is no significant difference between results produced by both algorithms.

Table 2. Wilcoxon Signed-Rank Test

	R^+	R^-
Original SKF vs Random QR SKF	171.0	264

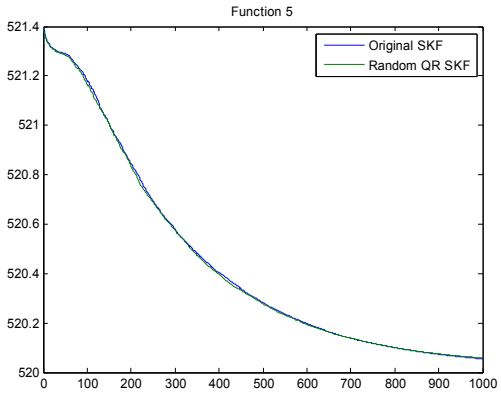

Fig. 1. Convergence curve for function 5.

4. Conclusion

Parameter tuning of an optimization algorithm could be a tedious task, especially, if the algorithm is employed to solve a specific problem. This study proves that *Q* and *R* tuning of SKF can be avoided, which is important towards the parameter-less SKF.

Acknowledgements

This research is supported by a Fundamental Research Grant Scheme (FRGS) awarded to Universiti Malaysia Pahang (RDU160105).

References

1. Z. Ibrahim, N. H. Abdul Aziz, N. A. Ab. Aziz, S. Razali, M. I. Shapiai, S. W. Nawawi, M. S. Mohamad, A Kalman filter approach for solving unimodal optimization problems, *ICIC Express Letters.* **9**(12) (2015) 3415-3422.
2. D. Goldberg, *Genetic Algorithms in Search, Optimization, and Machine Learning* (Reading, Addison-Wesley Professional, MA, 1989).
3. J. Kennedy, R. Eberhart, Particle swarm optimization, in *Proc. IEEE Int. Conf. Neural Networks* (1995), pp. 1942–1948.
4. E. Rashedi, H. Nezamabadi-pour, S. Saryazdi, GSA: a gravitational search algorithm, *Information Sciences.* **179** (13) (2009) 2232-2248.
5. Z. Md. Yusof, Z. Ibrahim, I. Ibrahim, K. Z. Mohd Azmi, N. A. Ab. Aziz, N. H. Abd Aziz, M. S. Mohamad, Angle modulated simulated Kalman filter for combinatorial optimization problems, *ARPN Journal of Engineering and Applied Science.* **11**(7) (2016) 4854-4859.
6. Z. Md. Yusof, I Ibrahim, S. N. Satiman, Z. Ibrahim, N. H. Abd Aziz, N. A. Ab. Aziz, BSKF: binary simulated Kalman filter, in *Proc. 3rd Int. Conf. Artificial Intelligence, Modeling and Simulation* (Sabah, Malaysia, 2015), pp. 77–81.
7. Z. Md. Yusof, Z. Ibrahim, I. Ibrahim, K. Z. Mohd Azmi, N. A. Ab. Aziz, N. H. Abd Aziz, M. S. Mohamad, Distance evaluated simulated Kalman filter for combinatorial optimization problems, *ARPN Journal of Engineering and Applied Science.* **11**(7) (2016) 4904-4910.
8. B. Muhammad, Z. Ibrahim, K. H. Ghazali, K. Z. Mohd Azmi, N. A. Ab. Aziz, N. H. Abd Aziz, M. S. Mohamad, A new hybrid simulated Kalman filter and particle swarm optimization for continuous optimization problems, *ARPN Journal of Engineering and Applied Science.* **10**(22) (2015) 17171-17176.
9. B. Muhammad, Z. Ibrahim, K. Z. Mohd Azmi, K. H. Abas, N. A. Ab. Aziz, N. H. Abd Aziz, M. S. Mohamad, Four different methods to hybrid simulated Kalman filter (SKF) with gravitational search algorithm (GSA), in *Proc. 3rd National Conf. Postgraduate Research* (Pekan, Malaysia, 2016), pp. 854–864.
10. K. Z. Mohd Azmi, Z. Md. Yusof, S. N. Satiman, B. Muhammad, S. Razali, Z. Ibrahim, N. A. Ab. Aziz, N. H. Abd Aziz, Solving airport gate allocation problem using angle modulated simulated Kalman filter, in *Proc. 3rd National Conf. Postgraduate Research* (Pekan, Malaysia, 2016), pp. 875–885.
11. A. Adam, Z. Ibrahim, N. Mokhtar, M. I. Shapiai, M. Mubin, I. Saad, Feature selection using angle modulated simulated Kalman filter for peak classification of EEG signals, *SpringerPlus.* **5**(1580) (2016).