# High Survivability of a Large Colony Through a Small World Relationship 

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#### Abstract

In this paper, energy tophallaxis, distributed autonomous energy management methodology inspired by social insects and bat behaviour, and its advantage is shown by a series of computer simulations to address the survivability of organized agents group under dynamics environment with uncertainty. Uncertainty of organizational agent's behaviour is represented by 2 Lévy distributions. By controlling energy donation behaviour based on these distributions carefully, the survivability of a larger group that traditional works cannot analyze is examined. As a result, a only small friendship over organization makes the group's survivability improved dramatically.


## 1 INTRODUCTION

This paper proposes a basic methodology for resource assingnment problem in a multiple agents system with uncertainty. There is a lot of papers which discuss resource assingnment problem under conditions of complete infomation. Meanwhile, growing in popularity of Internet and robotic technology on our daily life has been accompanied by a marked difficulty of its efficient management. For example, let us think about the energy management of many mobile robots. Usually, their batties are charged by being plugged in. When the number of plug is restricted or there is not enough time to be charged, how should we manage them? It is not easy question because of uncertainty. Of course, the group of robots are desiged carefully for some specific tasks but their behaviour is not $100 \%$ known beforehand. Especially, it seems to be very difficult to predict their energy consumption when they
engage in jobs to interact directly with human. As we know, human changes its mind frequently. However, even if there is this innate difficulty, it is undeniable fact that some methodology that can design resource allocation under such uncertainty is required.

We have focused attention on trophallaxis. Trophallaxis is mouth-to-mouth food sharing among ants[1], bats[5], and other some social animals. Basically, traditional energy management of robots have been based on nest like "central sharing", that is difficult to share their energy by congestion. On the other hand, this new energy sharing strategy, energy trophallaxis, can provide more flexible energy flow so that they can survive longer and stable[3][2].

In this paper, we discuss the survivability of a arger group with trophallaxis type energy sharing. When the colony is small, the hypothesis that each agent can transfer its energy to any member directory is reasonable, because the travel cost for reaching a recipient is negligible. Therefore, when the feeding success rate, the probability of getting energy from the environment, is independent, the larger the colony, the easier it is for its members to survive [3][6].

However, in general, it is difficult to assume this condition when the colony is large because some agent pairs must pay considerable travel costs. We show that this difference is sufficiently critical to result in members in larger colonies having a shorter life. By simulation, we show that an increase in the colony size makes it difficult for agents in a common organization to survive. Despite this counterproductive scale effect, we show that trophallaxis is still a good strategy for a large colony to achieve high survivability, by the law of large numbers. The scale effect can be solved by small world trait, which is normally found in ordinary


Figure 1: An example of high correlation between behaviour in the workspace and the organization.
relationships with friends.
This paper is composed as follows. In the next section, we propose our approach for handling large colony sizes. Two static networks are introduced: organization network and friend network. In our framework, agents meet others statistically. We suppose that all agents belong to the same organization, and an agent meets others according to a rendezvous probability, based on the topological distance of the organization network. When a pair of agents meet, the richer decides whether it donates energy to the poorer according to a permission probability, based on the topological distance of the relationships with friends in the friend network. Using these two probability distributions and networks, we can examine a variety of organizational structures.

In section 3, we briefly explain the trophallaxis model. We proposed an extension of the vampire bat energy model [6]. This model is useful because it is simpler than those of other related works [2][3].

In section 4 , using simple computer simulation, the emerging properties are discussed.

## 2 THE ORGANIZATIONAL STRUCTURE

### 2.1 Organizational network

This paper examines a large colony. In this case, the travel cost for some agent pairs is not negligible, so we assume that energy transfer is conducted when a pair of agents happen to meet. When the agents rendezvous, one decides whether to donate energy to the other.

We suppose that the rendezvous frequency depends


Figure 2: Networks used as organizational structures :(a) Complete Graph, (b) Regular Graph, (c)Beta Graph, (d) Random Graph
on the relationship of the agent pair on the organization of their colony. Obviously, the frequency of rendezvous is also determined by behaviour, however, in this paper, we are not interested in particular practical robot behaviour and tasks. Therefore, we introduce colony organization, rather than some concrete behaviour to approximate the frequency of rendezvous. When a colony is well organized, the organizational structure seems to provide a strong correlation between the working area of the members and the topological relationship of the organization, as shown in Fig.1. This figure shows a robot system that the designer wants to realize. The agents on each floor must work cooperatively, so these agents will have strong relationships. Obviously, these agents will meet frequently, therefore, the designer will assign the appropriate number of robots to build an effective robotic system. When a well-organized structure is provided, it is reasonable that the frequency of rendezvous can be estimated from the topological distance of the organization structure.

We assume that this each rendezvous takes place following by a probabilistic function based on their relative distances within the organizational structure. The organizational structure is represented by a graph with undirected links. Each node represents an agent. We suppose that the distance is the shortest link distance in the network. Also, the network is static, and there are no changes during trials. For example, if an agent is completely exhausted, the corresponding agent's node is not alive. However, this void node does not change the shortest distance calculation for


Figure 3: Shortest distance distribution of the networks adopted in this paper.
other agents. Note that two different networks are introduced - one for the rendezvous and the other is for permission probability of donating. However, both networks use the same notation and distance calculation method. Later, we describe the details of the network for judging whether to donate. The organization network $\mathbf{S}$ gives its adjacency matrix $\mathbf{A}$ of $C$. Now, when a colony $C=\{i \in\{1, \cdots, n\}\}$ is given, if there is a link between agent $i$ and $j$, the element $a_{i j}$ of $\mathbf{A}$ is 1 , otherwise it is 0 .

The distance between a pair of agents is calculated as the shortest distance of $\mathbf{A}$. Let the distance of agent $i$ and $j$ be $d(i, j)(=d(j, i))$. The shortest distance is then deduced by the Dijkstra method.

This paper adopts a complete graph, a lattice graph (a kind of regular graph), a random graph, and a $\beta$-graph[4], as shown in Fig.2. Figure 3 illustrates the shortest distance distribution of these graphs for $n=200$.

The complete graph corresponds to a small, well organized structure. Each agent has links to all others (Fig.2a), therefore, the distance from any agent to any other member is 1 , which is shortest. This suggests that they all work together in a vicinity.The lattice graph is introduced to represent organizations in which the members interact well locally. As shown in Fig.2b, each agent has $L$ equal number of links to its neighbours. When $L=2$, they connect as a ring. This means that there is a less well marked global structure, but there are strong relationships among neighbours. Figure 3 shows the shortest distance distribution for $L=4$. There are a constant number of agents located in a range from very close to very far away. The random graph represents an unorganized organization structure (Fig.2d). The distance distribution is similar to a normal distribution. The $\beta$-graph [4] is generated by applying the random rewiring procedure to the lattice graph noted above. A link of the lattice graph, se-


Figure 4: Rendezvous probability of $\alpha$
lected by probability $p$, is removed and reconnected to another node. This process is called random rewiring. Even if this probabilistic procedure causes the network to lose strong connectivity, there is no special compensation algorithm. Obviously, the graph with the random rewiring probability of $p=0$ is the original lattice graph, and the graph with a random rewiring probability of $p=1$ is same as the random graph. This parameter changes the characteristics, in particular for the graph with a probability of around $p=0.1$ is known, as it has small world characteristics[4]. This property means that the average distance is slowly increased as $\log (n)$. In Fig.3, the $\beta$-graph is made from an $L=4$ regular graph, and $p=0.1$ is shown. The degree distribution at $L=4$ is like that of the original lattice graph, whereas the distance distribution is completely different from the original.

### 2.2 Rendezvous probability based on the organization network

Let the distance of agent $i$ and $j$ in a given organization network be $d_{o}(i, j)\left(=d_{o}(j, i)\right)$. Now we suppose that the rendezvous probability is defined as the power function fo of distance $d_{o}(i, j)$ as follows.

$$
\begin{equation*}
f\left(d_{o}(i, j)\right)=m / d_{o}(i, j)^{\alpha_{o}} \tag{1}
\end{equation*}
$$

In the remainder of this paper, $m=0.5 . \alpha_{o}$ is a control parameter. This function has been well studied in many areas, for example, the Lévy flight probabilistic procedure. In Fig.4, the probability distributions $\alpha_{o}=\{2.0,2.5,3.0\}$ are shown. If $\alpha_{o}$ is small $\left(\alpha_{o} i 2\right)$, agents too far away still have a high probability of meeting; when $\alpha_{o}$ is large, for example, $\alpha_{o} ¿ 3.0$, agents only meet their closer neighbours.

## 3 ENERGY SHARING MODEL AND ITS EVALUATION

### 3.1 The Vampire Bat Energy Sharing Model

This section explains the trophallaxis energy exchange procedure. When a pair of agents meet according to the rendezvous probability in eq.1, they have a chance to exchange energy. We adopt a model proposed by [6] for this exchange procedure. This model is simple and well-grounded because it simulates the energy exchange procedure of the common vampire bat, which is a famous example of for trophallaxis [5].

The essence of the model is that each agent tries to get food once every 24 steps. The feeding success probability is called the feederate. Now, if agent $i$ succeeds, its energy $e_{i}$ is fully increased.

$$
\begin{equation*}
e_{i}(t)=e_{\max } \tag{2}
\end{equation*}
$$

Each agent consumes ecycle per step. If $e_{i}<0$, it is dead. If $e_{i}<e_{\text {need }}$, agent $i$ requests donations from richer agents - those who have more energy than $e_{\text {have }}$. If a rich agent accepts the request, the recipient receives thave energy per donation. The exchange loss during transfer is represented by $e_{\text {efficiency }}$. For example, if $e_{\text {efficiency }}=100$, the energy which a donor loses is the same as that obtained by the recipient; if the recipient receives no energy, then $e_{\text {efficiency }}=0$. That is,

$$
\begin{gather*}
e_{\text {donor }}(t+1)=e_{\text {donor }}(t)-t_{\text {have }}  \tag{3}\\
e_{\text {recipient }}(t+1)=e_{\text {recipient }}(t)+t_{\text {have }} e_{\text {efficiency }} / 100 \tag{4}
\end{gather*}
$$

These parameters are set as follows: $e_{\text {cycle }}=1$, $e_{\text {max }}=60, \quad e_{\text {need }}=24, \quad e_{\text {have }}=28, \quad t_{\text {transfer }}=3$, $e_{\text {efficiency }}=80$, feederate $=0.7$. These are reflected in the vampire bat. Under this parameter set, a donor cannot be a recipient after single donation because

$$
\begin{equation*}
e_{\text {have }}-t_{\text {transfer }}>e_{\text {need }} \tag{5}
\end{equation*}
$$

Therefore, this is called the stable condition.

### 3.2 Permission probability on the friend network and donor selection

When an agent is starving ( $e<e_{\text {need }}$ ), it makes havelisti which is a set of donor candidates.

As mentioned previously, two networks are introduced in this paper: the organization network, which
controls the rendezvous probability, and the friend network. The judgment as to whether an agent donates energy when a pair of agents meet is determined by the permission probability based on the distance between them in this network. The notation of the friend network is same as for the organization network. Also, the same network types are adopted.

Let the distance of agent $i$ and $j$ in the friend network be $d_{f}(i, j)\left(=d_{f}(j, i)\right)$. Now we suppose that the permission probability is also defined as the power function $f_{f}$ of distance $d_{f}(i, j)$ as follows.

$$
\begin{equation*}
f\left(d_{f}(i, j)\right)=m / d_{f}(i, j)^{\alpha_{f}} \tag{6}
\end{equation*}
$$

In the rest of this paper, $m=0.5 . \alpha_{f}$ is a control parameter. When a pair of agents meet, if one is starving and the other has enough energy to satisfy the stable condition (see eq.5), the richer one joins the recipient's havelist with the probability $f_{f}\left(d_{f}(i, j)\right)$.

When there is more than one rich agent ( $e>e_{\text {have }}$ ) in a recipient's havelist, their order can cause another problem. In this model, a donor is selected randomly. The trophallaxis is executed repeatedly until there are no starving agents or there are no rich agents in the starving agent's havelist.

### 3.3 Evaluation criteria:survivability

In this model, survivability is employed as the evaluation criterion. Each simulation is executed for $10 \times 365 \times 24$ steps, which corresponds to 10 years - a sufficiently long span. The ratio of survivors to the number of initial members is evaluated as the survivability.

## 4 COMPUTER SIMULATION

In this section, we show the characteristics of a large size colony by employing colony size $n$, the rendezvous probability of $\alpha_{o}$, based on the organization network, and the permission probability of $\alpha_{f}$, based on the friend network.

### 4.1 Complete graph organization with trophallaxis

First, we conduct a simulation to address the affect of scale on trophallaxis. We use four somewhat small different sized colonies with trophallaxis $n=\{15,20,25,35\}$, and a colony of size $n=35$ without trophallaxis. All colonies with trophallaxis adopt the complete graph as their organization and friend


Figure 5: Survivability of a small colony.


Figure 6: Size effect of lattice organization colony.
networks, so that a member can get energy from any other member. Each colony is examined for at least 50 trials and the average survivability for each day is shown in Fig.5.

The dotted line indicates the survivability of the colony of 35 agents without trophallaxis. The x axis is the time (day=step/24) and the y axis is the survivability. As you see, survivability drops quickly. There was no trial in which at least one agent survived over 150 days.

The solid lines illustrate the results for the colonies with trophallaxis. When $n=15$, the colony can survive longer than the colony without trophallaxis but for no trial among 20 trials did at least one agent survive more than 1250 days. However, colonies larger than 30 agents can survive over 10 years.

This result can be understood as follows. In this exchange model, each individual's feeding success rate is independent and this model assumes that the total amount of food in the environment is unlimited. In this case, the number of agents who get food converges at $n \cdot f$ eederate when $n$ becomes infinite, by the law of large numbers theorem. Consequently, the probability of the occurrence of the "less food" state goes to zero. Therefore, we can say that trophallaxis offers a great advantage for survival if agents can exchange energy with any other agent at any time.

### 4.2 Lattice graph organizations with trophallaxis

In the previous subsections, we conducted a set of experiments to address the survivability of trophallaxis of ideal colony. In this successive subsections, lattice networks with $L=20$ for a more practical large colony are used mainly. In section 4.2 .2 , the organizational structure adopts a lattice graph and a friend network that is a $\beta$-graph network, based on the lattice graph.

### 4.2.1 Lattice graph friend network with lattice organization

When a pair of agents meet and one is able to donate energy, first the donor candidate decides whether it will donate, based on its permission probability $f_{f}(d)$ (see eq.6.). The parameter $\alpha_{f}$ controls the range of the donation. When $\alpha_{f}$ is smaller than $\alpha_{o}$, in eq.1, the agent gives energy agents who it rarely meets. If $\alpha_{f}$ is larger than $\alpha_{o}$, however, it does not donate to agents who it meets frequently. First, we conduct a simulation to bring out the effect of the colony size, $n$. We used 15 colonies by combining colonies of different sizes, $n=\{25,50,75,100,150\}$ and three different organization networks $\alpha_{o}=\{2.0,2.5,3.0\}$. The friend network parameter $\alpha_{f}$ of all of these is 2.5. Figure 6 shows the survivability. When the colony size is small ( $\mathrm{n}=25$ ), they partially survive. Survivability improves with an increase of $n$ until some limit. After the limit, it becomes worse, depending on $\alpha_{o}$.

We think this result is very important because it indicates that the colony size has an adverse affect on survival. Even if the feeding success rate is independent, the survivability becomes worse as the colony size increases, when the organizational structure is a lattice graph.

The reason could be that, in such an organization, every agents has 20 links to their neighbours. Therefore, a donor can give its energy to a recipient 10 units away from it. Obviously, the speed of the diffusion of energy in this case is faster than for a network with a smaller number of links, for example $L=2$ or 3 . Thus, surplus energy moves away from its donor quickly. Therefore, in the long term, its donation does not make its neighbour and itself rich. When the colony size is not too large, namely $n=50$, as shown in Fig.6, we suppose that the diffused energy comes back before it vanishes due to exchange losses, $e_{\text {efficiency }}$. because it takes few exchanges to make a circuit of the organization. Therefore, survivability is improved. When


Figure 7: $\beta$-graph friend network over a dense lattice organization ( $\alpha_{o}=2.0$ ).


Figure 8: $\beta$-graph friend network over a sparse lattice organization ( $\alpha_{o}=2.5$ ).
the colony is too large, the scale effect is lost. More work is needed to clarify this issue.

### 4.2.2 $\beta$-graph friend network over a lattice organization

The last section showed that trophallaxis does not work well in a large lattice graph organizational structure. We suggested the reason is the quick diffusion of energy. In this section, we conduct a simulation addressing this trend. In generally speaking, a $\beta$-graph with a small rewiring probability has high clustering coefficient. This means that there are many small triangles in the organization. Therefore, it is reasonable to ask whether this helps retain surplus energy.
Figures 7 and 8 show the survivability of a $\beta$-graph friend network with $L=20$ over lattice graph organization network with $L=20, n=100$, and $\alpha_{o}=\{2.0,2.5\}$. In both graphs, the x axis is the rewiring probability.

The effect is impressive. Of all these situations, the colony with a small rewiring probability, $p=0.125$, can survive more consistently than that of one with a rewiring probability of $p=0.0$. Although more work is require to clarify the details of this mechanism, this high survivability ensures that we can say that the rewiring procedure can produce successful trophallaxis.

## 5 CONCLUSION

In this paper, the survivability of large colonies with trophallaxis energy transfer is examined by introducing two networks: an organization, and a friend network. In a large colony, the travel cost for some agent pairs is not negligible, so that energy transfer is conducted when a pair of agents happen to meet. The probability of a rendezvous is determined by the organization network. After a pair of agents meet, one decides whether to donate energy, based on its permission probability, which is defined by the friend network. By this simplified approach, we can examine the characteristics of survivability for a large colony with trophallaxis, for example, the scale effects and unfavourable issues.

Several computer simulations produced the following observations: 1) In general, if the colony is small, survivability improves with an increase of members. However, even if their feeding success rate is independent, their survivability becomes worse with an increase in colony size when their organization structure is a lattice graph; and, 2) it is possible to deal with this unfavourable characteristic using the random rewiring procedure of $\beta$-graph, which yields a small world phenomenon.

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